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Optimal hybrid reinsurance strategy based on the classic model of minimizing ruin risk

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Introduction

In real life, the amount of insurance claims is random, and huge claims may put the insurance company in a predicament of being unable to bear the claims and on the verge of bankruptcy. At this time, a reasonable reinsurance contract becomes particularly important. The issue of optimal reinsurance was first proposed by Borch in 1960, who derived the optimal reinsurance strategy based on the maximization of wealth utility. Since then, more and more scholars have begun to study the issue of optimal reinsurance, mainly from the following aspects.

- Considering the differences in reinsurance strategies.
- Considering the differences in premium principles.
- Considering the different optimization criteria for optimal reinsurance problems.
- Considering that pure reinsurance strategies cannot meet complex social needs.

Hybrid reinsurance model

The Lagrangian function is obtained as follows:

$$\begin{aligned}L(\theta,q,s,\xi) &= Var(W_1+W_2)+\xi[E(R(\theta,q,s))-k]\\ &= \theta^2q^2\lambda\int_0^\infty x^2dF(x)+(1-\theta)^2\lambda[\int_0^s x^2dF(x)+\int_s^\infty s^2dF(x)]\\ &+2\theta(1-\theta)q\lambda[\int_0^s(s-2x)F(x)dx+s\mu]\\ &+\xi[\alpha\lambda\int_0^\infty x^2dF(x)-\theta\beta(1-q)^2\lambda\int_0^\infty x^2dF(x)\\ &-(1-\theta)\gamma\lambda\int_s^\infty(x-s)^2dF(x)-m],\end{aligned}$$

The Lagrange equation for (θ,q,s) are

$$\begin{aligned}\frac{\partial L}{\partial\theta} &= 2\lambda\theta q^2\int_0^\infty x^2dF(x)-2\lambda(1-\theta)[\int_0^s x^2dF(x)+\int_s^\infty s^2dF(x)]\\ &+(1-2\theta)2\lambda q[\int_0^s(s-2x)F(x)dx+s\mu]\\ &+\xi[\gamma\lambda\int_s^\infty(x-s)^2dF(x)-\beta(1-q)^2\lambda\int_0^\infty x^2dF(x)]=0\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial q} &= 2\theta^2q\lambda\int_0^\infty x^2dF(x)+2\theta(1-\theta)\lambda[\int_0^s(s-2x)F(x)dx+s\mu]\\ &+2\xi\beta\theta\lambda(1-q)\int_0^\infty x^2dF(x)=0,\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial s} &= 2s\lambda(1-\theta)^2(1-F(s))+2\lambda\theta(1-\theta)q[\int_0^s F(x)dx-sF(s)+\mu]\\ &+2\xi(1-\theta)\gamma\lambda\int_s^\infty(x-s)dF(x)=0,\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial\xi} &= \alpha\lambda\int_0^\infty x^2dF(x)-\theta\beta(1-q)^2\lambda\int_0^\infty x^2dF(x)\\ &-(1-\theta)\gamma\lambda\int_s^\infty(x-s)^2dF(x)-m=0.\end{aligned}$$

By solving the equations consisting of equations, we can obtain the optimal solution satisfied by the parameters (θ,q,s).

Numerical example

Assume that the insurance claim amount $\{X_k, k=1,2,\cdots\}$ discussed in this example follows an exponential distribution with parameter 1. Let the safety load α of the original insurance company be 0.2, the safety load of the proportional reinsurance company and the excess loss reinsurance company be 0.3, and the claim counting process $\{N(t), t\geq 0\}$ be a Poisson process with a Poisson distribution intensity λ of 1.

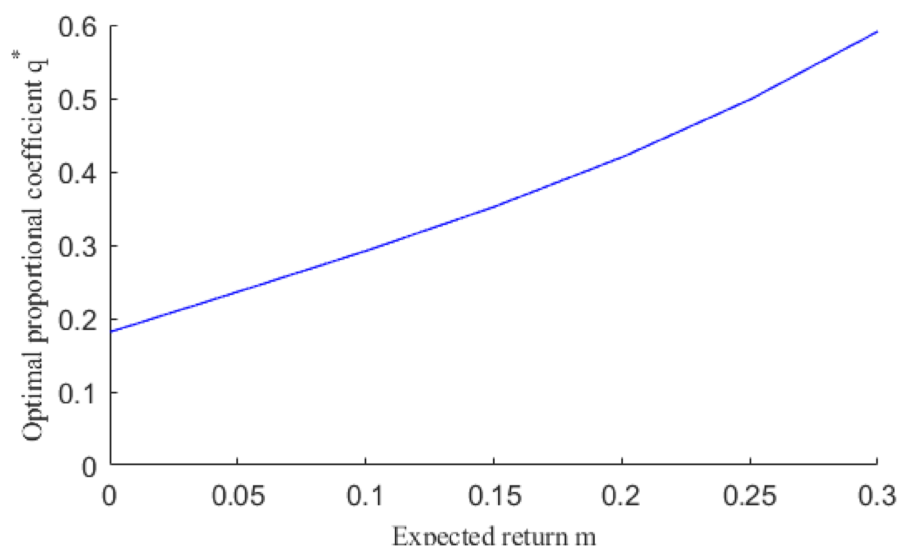

Figure 1. The relationship between expected return and optimal ratio coefficient

When the Poisson distribution strength λ of the original insurance company is 1 and 2

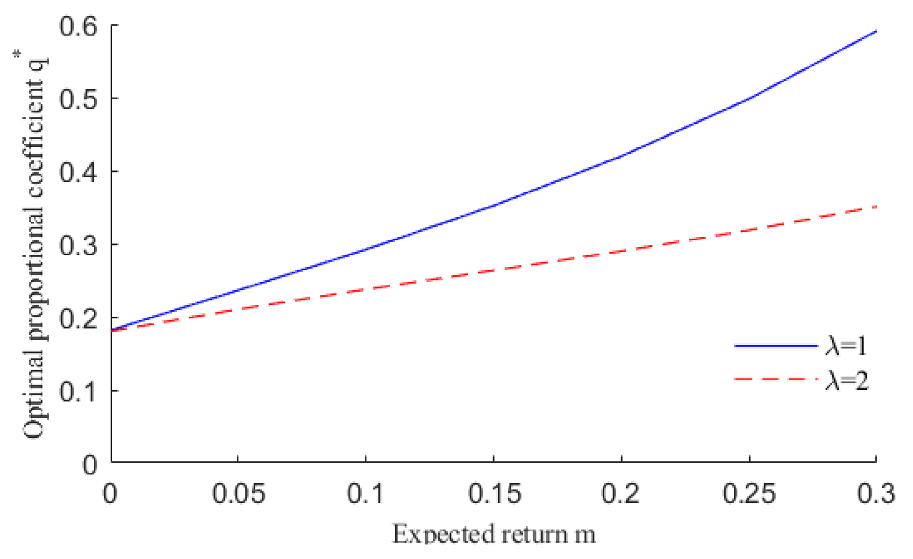

Figure 2. The relationship between expected return and optimal proportionality coefficient under different Poisson distribution intensities

When the proportional reinsurance safety load β is set to 0.3 and 0.31

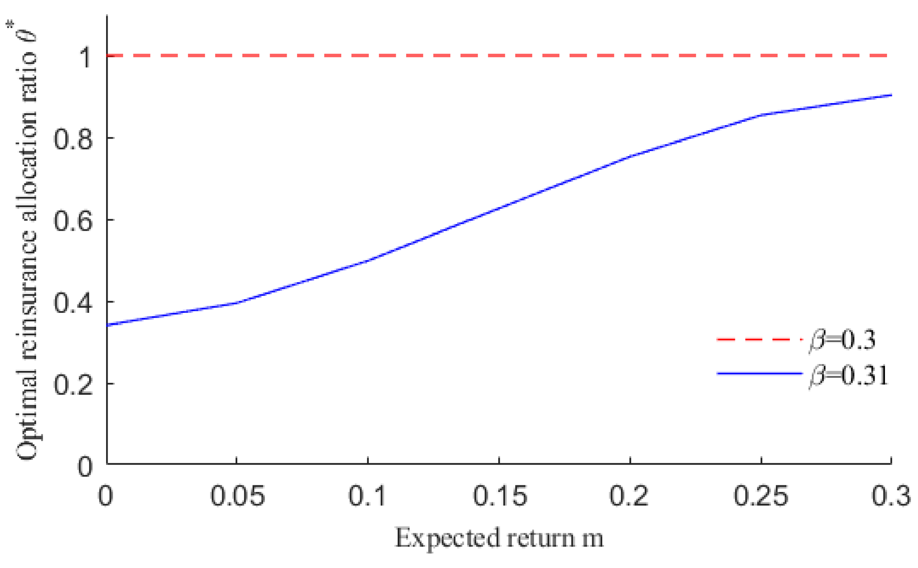

Figure 3. Relationship between expected return and optimal reinsurance allocation ratio under different safety loads

Result

Therefore, when faced with different Poisson distribution intensities, different reinsurance company safety loads, and different claim insurance types, insurance companies can bring more substantial development by reasonably changing their own operating parameters. This example specifically illustrates that the study of the hybrid reinsurance contract form of proportional reinsurance and excess loss reinsurance has important practical significance.

Conclusions

- This paper extends the pure proportional reinsurance strategy and excess loss reinsurance strategy into a hybrid reinsurance s trategy.
- This paper calculates the premiums of the original insurance company and the reinsurance company based on the variance premium principle rather than the expected value premium principle.

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