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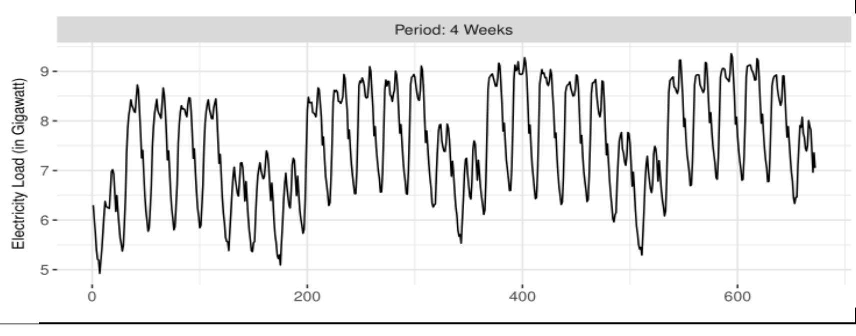
Identification of Triple Seasonal Autoregressive Models: A Bayesian Approach

Ayman A. Amin
Department of Statistics, Mathematics, and Insurance,
Faculty of Commerce, Menoufia University, Menoufia, Egypt

Introduction

High frequency time series are observed in real applications, such as the hourly electricity load, that exhibit multiple seasonalities, as in the figure.



Triple seasonal autoregressive (TSAR) models were introduced to accommodate these multiple seasonalities. TSAR models' identification is the initial and most-important stage for analyzing time series with triple-seasonality in real applications.

Research Objective

Introducing an efficient Bayesian identification method to specify the best TSAR model order.

TSAR Models

Time series can be modeled by the TSAR model with order p, P_1, P_2 and P_3, shortened as $\text{TSAR}(p)(P_1)_{s_1}(P_2)_{s_2}(P_3)_{s_3}$ and written as:

$$\theta_p(B)\vartheta_{P_1}(B^{s_1})\phi_{P_2}(B^{s_2})\varphi_{P_3}(B^{s_3})w_t = u_t \qquad (1)$$

These models can be written in the matrix form as:

$$\mathbf{w} = \mathrm{H}\Gamma + \boldsymbol{u} \qquad (2)$$

TSAR Bayesian Concepts

The **conditional-likelihood function**:

$$L(\Gamma,\tau|w) \quad \propto \quad \tau^{\left(\frac{n-q}{2}\right)}\exp\left\{-\frac{\tau}{2}(\mathbf{w}-\mathbf{H}\Gamma)^T(\mathbf{w}-\mathbf{H}\Gamma)\right\} \qquad (3)$$

The **normal-gamma prior** of Γ and τ as:

$$\zeta_n(\Gamma,\tau) \quad \propto \quad \tau^{\left(\frac{\nu+p^\star}{2}-1\right)}\exp\left\{-\frac{\tau}{2}\left[\lambda+(\Gamma-\mu_\Gamma)^T\Sigma_\Gamma^{-1}(\Gamma-\mu_\Gamma)\right]\right\}, \qquad (4)$$

Jeffreys' prior on Γ and τ as:

$$\zeta_j(\Gamma,\tau) \quad \propto \quad \tau^{-1}, \quad \tau > 0 \qquad (5)$$

Prior information about the TSAR order p, P_1, P_2 and P_3 in term of $\zeta(p,P_1,P_2,P_3)$ in different forms:

$$\zeta_1(p,P_1,P_2,P_3) \quad = \quad \frac{1}{V_1}\times\frac{1}{V_2}\times\frac{1}{V_3}\times\frac{1}{V_4}, \qquad \text{(Uniform)}$$
$$\forall p=1,\ldots,V_1, P_1=1,\ldots,V_2, P_2=1,\ldots,V_3, P_3=1,\ldots,V_4 \quad (6)$$

$$\zeta_2(p,P_1,P_2,P_3) \quad \propto \quad \frac{V_1-p+1}{V_1+1}\times\frac{V_2-P_1+1}{V_2+1}\times\frac{V_3-P_2+1}{V_3+1}\times\frac{V_4-P_3+1}{V_4+1}, \quad \text{(Arithmetic)}$$
$$\forall p=1,\ldots,V_1, P_1=1,\ldots,V_2, P_2=1,\ldots,V_3, P_3=1,\ldots,V_4 \quad (7)$$

$$\zeta_3(p,P_1,P_2,P_3) \quad \propto \quad 0.5^{p+P_1+P_2+P_3}, \qquad \text{(Geometric)}$$
$$\forall p=1,\ldots,V_1, P_1=1,\ldots,V_2, P_2=1,\ldots,V_3, P_3=1,\ldots,V_4 \quad (8)$$

For the normal-gamma prior, we obtain **the joint posterior** of the TSAR parameters as:

$$\zeta_n(\Gamma,\tau,p,P_1,P_2,P_3|w) \quad \propto \quad \exp\left\{-\frac{\tau}{2}\left[\lambda+(\Gamma-\mu_\Gamma)^T\Sigma_\Gamma^{-1}(\Gamma-\mu_\Gamma)+(\mathbf{w}-\mathbf{H}\Gamma)^T(\mathbf{w}-\mathbf{H}\Gamma)\right]\right\} \times \tau^{\left(\frac{n-q+\nu+p^\star}{2}-1\right)} \times \zeta(p,P_1,P_2,P_3) \qquad (9)$$

Also, for Jeffreys' prior, we write **the joint posterior** as:

$$\zeta_j(\Gamma,\tau,p,P_1,P_2,P_3|w) \quad \propto \quad \tau^{\left(\frac{n-q}{2}-1\right)}\exp\left\{-\frac{\tau}{2}(\mathbf{w}-\mathbf{H}\Gamma)^T(\mathbf{w}-\mathbf{H}\Gamma)\right\}\zeta(p,P_1,P_2,P_3) \quad (10)$$

Suggested Bayesian Identification of TSAR Models

Theorem 1. Given the conditional-likelihood function (3) along with the normal-gamma prior distribution (4), the PMF of the TSAR order is obtained as:

$$\zeta_n(p,P_1,P_2,P_3|w) \propto \zeta(p,P_1,P_2,P_3)\left[\frac{|\Sigma_\Gamma^{-1}|}{|X_n|}\right]^{1/2}\left[w^Tw+\lambda+\mu^*+-Z_n^TX_n^{-1}z_n\right]^{-\frac{n-q+\nu}{2}}$$
$$\forall p=1,..,V_1, P_1=1,..,V_2, P_2=1,..,V_3, P_3=1,..,V_4. \qquad (11)$$

Where $\mu^* = \mu_\Gamma^T\Sigma_\Gamma^{-1}\mu_\Gamma$ $X_n = (H^TH+\Sigma_\Gamma^{-1})$, and $Z_n = (H^T\mathbf{w}+\Sigma_\Gamma^{-1}\mu_\Gamma)$.

Corollary 1. Given the conditional-likelihood function (3) along with Jeffreys' prior distribution (4), the PMF of the TSAR order is obtained as:

$$\zeta_n(p,P_1,P_2,P_3|w) \propto \zeta(p,P_1,P_2,P_3)\frac{\Gamma\left(\frac{n-q^*}{2}\right)}{\pi^{\frac{n-q}{2}}|H^TH|^{1/2}}\left[w^Tw-w^TH(H^TH)^{-1}H^Tw\right]^{-\frac{n-q-p^\star}{2}}$$
$$\forall p=1,..,V_1, P_1=1,..,V_2, P_2=1,..,V_3, P_3=1,..,V_4. \qquad (12)$$

TSAR order is identified by calculating posterior probabilities from the joint PMF for all possible values, and then choosing the value with the highest posterior probability to be the best TSAR order.

Simulation Study

Generating 500 of time series from two TSAR models, with a sample size n ranges from 3,000 to 6,000.

TSAR Model	ϕ_1	ϕ_2	Φ_1	Φ_2	Π_1	Π_2	Ψ_1	τ
I. $(1)(1)_{12}(1)_{60}(1)_{600}$	0.9		0.7		0.8		0.7	1.0
II. $(2)(1)_{12}(1)_{60}(1)_{600}$	0.6	0.3	0.9		-0.8		0.7	1.0

	Model I		Model II	
n	PMF	Testing	PMF	Testing
3000	87.8	85.5	97.6	84.0
4000	92.0	87.8	99.0	89.2
5000	94.8	87.4	99.2	89.6
6000	95.8	88.0	99.6	89.0

- Larger size the higher percentage of correctly identified true TSAR models is obtained.
- Various priors result in slightly different percentage of correctly identified true TSAR models.
- Suggested Bayesian identification is robust against the slight violation of normality assumption.

	Model I			Model II		
n	U	A	G	U	A	G
TSAR model with normal errors						
3000	87.8	88.6	89.4	97.6	97.6	97.6
4000	92.0	92.8	93.4	99.0	99.2	99.2
5000	94.8	95.4	95.6	99.2	99.2	99.2
6000	95.8	96.4	96.4	99.6	99.6	99.6
TSAR model with t(5) errors						
3000	88.6	89.4	90.4	95.0	95.6	95.6
4000	92.2	92.8	93.6	96.8	97.0	97.2
5000	94.4	94.4	95.2	97.4	97.4	97.6
6000	96.0	96.4	97.0	97.8	97.8	97.8

Conclusion

- An efficient Bayesian identification method for TSAR models is proposed to fit time series with the triple-seasonality.
- Extensive simulations confirmed the accuracy of proposed Bayesian identification.
- The proposed Bayesian identification is more accurate than the existing testing-based identification procedure.

References

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