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Numerical solution method for second kind Fredholm integral equations based on SSA

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Introduction

Integral equations are widely applied in multiple fields, but their analytical solutions are often difficult to obtain and numerical methods are needed. Regarding the numerical solutions of the second kind of Fredholm integral equations, the existing research covers various techniques, such as the least squares support vector machine, Chebyshev neural network, BP neural network, Simpson and Gauss quadrature formulas, exponential transformation cosine expansion, adaptive wavelet neural network, collocation method, particle swarm algorithm, power series approximation, mean projection algorithm, Nystrom method and spline Gaussian rule, etc. However, these methods have-their own advantages and disadvantages in terms of solution accuracy and computational complexity.

Based on the previous studies, this paper innovatively adopts the Sparrow Search Algorithm to solve the second kind of Fredholm integral equations. The specific steps include: Selecting the power series as the approximate function, transforming the integral equation into an optimization problem,and then usingthe SparrowSearch Algorithmto find the optimal coefficients of the power series. Through comparison with the algorithms in the existing literature, the results of numerical examples show that the method proposed in this paper exhibits higher superiority in terms of solution accuracy. This research not only enriches the numerical solutions of Fredholm integral equations but also provides new ideas and technical means for solving similar problems.

MAIN BODY

1. The second kind of Fredholm integral equation

This paper investigates one-dimensional second kind of Fredholm integral equation, which has the following form:

$$u(x)-\lambda\int_a^b k(x,t)u(t)dt = f(x) \quad (1)$$

where $k(x,t)$ and $f(x)$ are given functions, $u(x)$ is the unknown function, λ is a parameter, and [a,b] are the integration intervals. The composite Simpson's rule is used to approximate the definite integral in equation (1). The integration interval [a,b] is divided into an even number 2m of subintervals with step size $(b-a)/n$, resulting in n+1 nodes.

2. SSA solves the second Fredholm integral equation

The Sparrow Search Algorithm (SSA) is a new type of swarm intelligence algorithm proposed in 2020, inspired by the collective intelligence, foraging, and anti-predator behavior of sparrows. In simulation experiments, virtual sparrows are used to search for food, and the location of each sparrow is represented by the equation (2) matrix. Where p represents the number of sparrows, and d represents the dimension of the variables. The fitness values of all sparrows can be represented using the f equation (3) matrix.

$$S=\begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1d} \\ s_{21} & s_{22} & \cdots & s_{2d} \\ \vdots & \vdots & \vdots & \vdots \\ s_{p1} & s_{p2} & \cdots & s_{pd} \end{bmatrix} \quad (2)$$

$$Fx=\begin{bmatrix} f(s_{11} & s_{12} & \cdots & s_{1d}) \\ f(s_{21} & s_{22} & \cdots & s_{2d}) \\ \vdots & \vdots & \vdots & \vdots \\ f(s_{p1} & s_{p2} & \cdots & s_{pd}) \end{bmatrix} \quad (3)$$

In this context, each row in Fx represents the fitness value of an individual. In SSA, the discoverers with better fitness values have priority in obtaining food during the search process. Discoverers are responsible for searching for food and guiding the movement of the entire population. The position update of discoverers in each iteration is as follows:

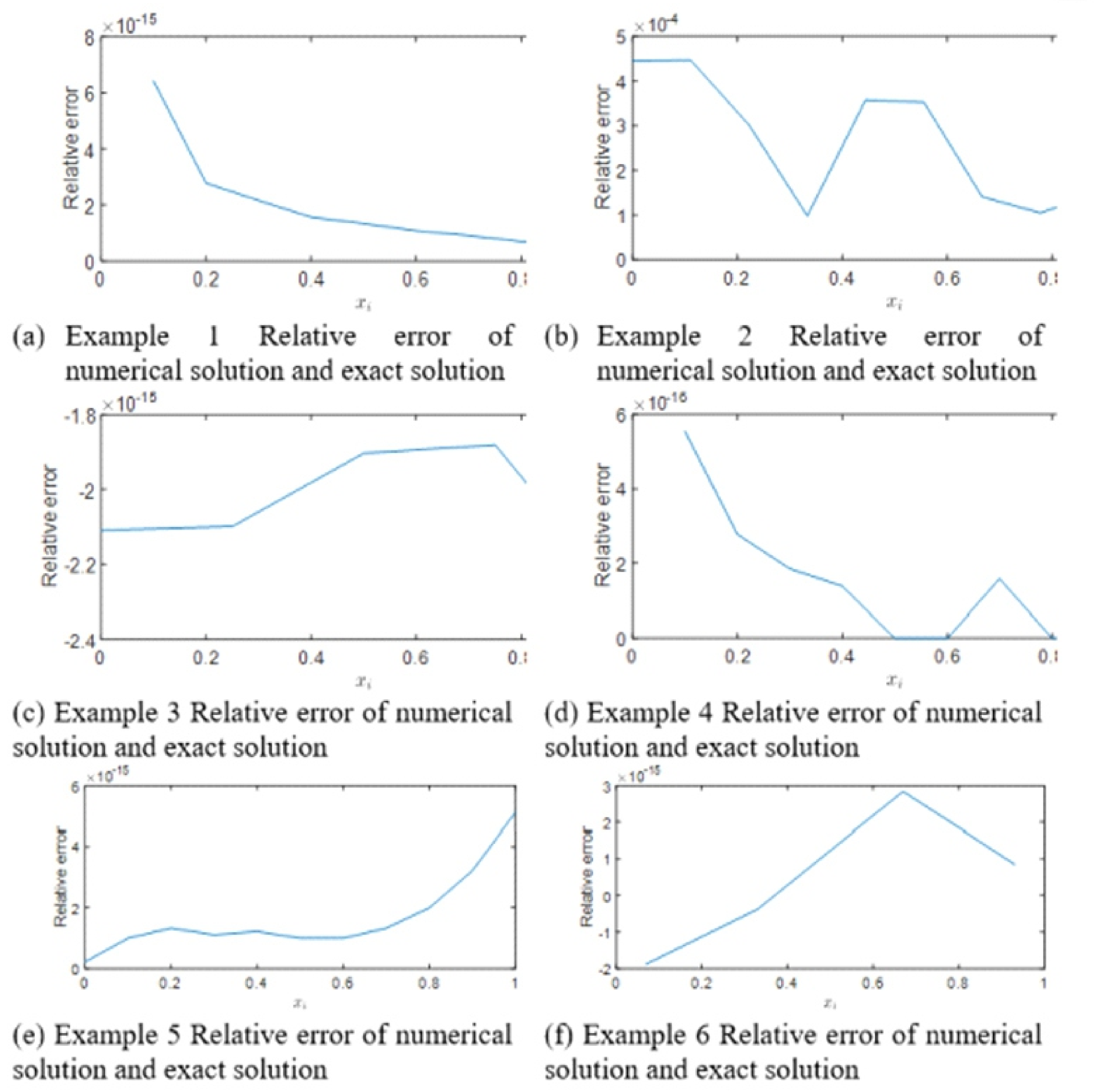
$$S_{i,j}^{t+1}=\begin{cases} S_{i,j}^{t}\times\exp\left(\dfrac{-i}{\alpha\times iter\max}\right) & if\ R_2<ST \\ S_{i,j}^{t}+Q\times L & if\ R_2\geq ST \end{cases}$$

Followers frequently monitor the discoverers. Once the discoverers find food, the followers immediately leave their current position to compete for the food, and their position update formula is:

$$S_{i,j}^{t+1}=\begin{cases} Q\times\exp\left(\dfrac{S_{worst}^{t}-S_{i,j}^{t}}{i^2}\right) & if\ i>n/2 \\ S_{op}^{t+1}+\left|S_{i,j}^{t}-S_{op}^{t+1}\right|\times A^{+}\times L & else \end{cases}$$

3. Numerical examples and analysis

To verify the effectiveness of the proposed method in this paper, all instances were run on a PC with an Intel(R) Core(TM) i7-9750H CPU, 2.60GHZ, and 8GB of memory using Matlab R2016a. The parameters were set as follows: population size of 50, maximum number of iterations of 1000, PD=0.2, ST=0.8, SD=0.2, power series , and the power series coefficient domain range of [-30,30]. The comparison of experimental results with existing literature is shown in Tables 1-7, and the fitting degree and relative (absolute) error between the numerical solution and the exact solution are shown in Figures 1-14.



(a) Example 1 Relative error of numerical solution and exact solution (b) Example 2 Relative error of numerical solution and exact solution (c) Example 3 Relative error of numerical solution and exact solution (d) Example 4 Relative error of numerical solution and exact solution (e) Example 5 Relative error of numerical solution and exact solution (f) Example 6 Relative error of numerical solution and exact solution

Figure 1. Relative error between numerical solution and exact solution graph.

Conclusion

In this paper, we have studied the numerical solution of second-kind linear and nonlinear Fredholm integral equations using the Sparrow Search Algorithm (SSA). This method first takes the power series as the approximate analytic expression of the unknown function, then converts the system of equations into an optimization problem, and finally uses the SSA to solve for the optimal power series coefficients. All experimental results demonstrate the feasibility, reliability, and superiority of the proposed method, providing a new approach for the numerical solution of second-kind Fredholm integral equations.