A weak-strong competition model with Robin and free boundary conditions

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Methods: Comparison principle, The maximum principle and Structural upper and lower solution

Introduction

Reaction diffusion equation is a kind of typical semi-linear parabolic partial differential equation, it can be derived from the process of the spread of invasive species, free boundary area is refers to the partial differential equation is unknown, need settlement is given, together with free boundary reaction-diffusion equation of research is one of the important direction of reaction diffusion equation of the research.

At present, Mathematicians have extensively studied competitive models with free boundary conditions. For example, Wang and Zhao [1,2] considered the one-dimensional reaction-diffusion competition model with Dirichlet and Neumann boundary conditions, proved that the alternative nature of invasive species expansion and disappearance was valid under strong-weak and weak-strong conditions, and gave an estimate of the asymptotic expansion rate of the free boundary during species expansion. Guo and Wu [3] studied the strong-weak case with Neumann boundary

The above studies are all aimed at Dirichlet and Neumann boundary conditions, while there are few researches on Robin boundary conditions, because the competition model with Robin boundary is more consistent with the species transmission process in some actual situations, which has theoretical and practical significance. Therefore, we mainly research the following Lotka-Volterra model with Robin and free boundary:

$$\begin{cases} p_t = p_{xx} + p(1-p-kq), & t>0, 0<x<g(t), \\ q_t = Dq_{xx} + \gamma q(1-q-hp), & t>0, 0<x<g(t), \\ p(t,0) = bp_x(t,0), q(t,0) = bq_x(t,0), & t>0, \\ p=q=0, \quad g'(t) = -\mu(p_x+\rho q_x), & t>0, \\ g(0)=g_0, & 0\le x\le g_0 \end{cases}$$

Results

Theorem 3.2 We assume that $g_\infty < \infty$, let $Q_0(x)$ be the only solution , then

$$\begin{cases} -DQ'' = \gamma Q(1-Q), & 0<x<g_\infty, \\ Q(0) = bQ'(0), Q(g_\infty)=0. \end{cases}$$

then

When $g_\infty \le R^*, \lim_{t\to\infty} \max_{0\le x\le g(t)} q(t,x) = 0;$

When

$g_\infty \ge R^*, \lim_{t\to\infty} \max_{0\le x\le g(t)} |q(t,x) - Q_0(x)| = 0.$

Theorem 3.3 Supposed that $g(\infty)=\infty$. if $0<h<1\le k$, then the solution $(p(t,x), q(t,x))$ of (1.1) meets

$$\liminf_{t\to\infty} p(t,x) \ge \underline{p}(x), \limsup_{t\to\infty} p(t,x) \le \overline{p}(x)$$

$$\liminf_{t\to\infty} q(t,x) \ge \underline{q}(x) \quad \limsup_{t\to\infty} q(t,x) \le \overline{q}(x)$$

consistently in $[0,M)$ $\forall M>0$.

Lemma 4.2 Suppose that $0<h<1\le k$, if $g_0 < \min\{R_1, R_2\}$, then there exist $\mu_0 > 0$, such that $s_\infty \le \infty$ provided $\mu \le \mu_0$.

Conclusion

When $g_\infty < \infty$ the inferior competitor p can not spread successfully as $t\to\infty$. While for the superior competitor q , there are two cases: One is when $g_\infty \le R^*$, q will die out eventually; the other is when $g_\infty > R^*$, q can spread successfully. However, when $g_\infty = \infty$, both p and q have upper and lower bounds. Moreover, we obtain the spreading and vanishing of criteria.